Applications of Number Theory and Algebraic Systems

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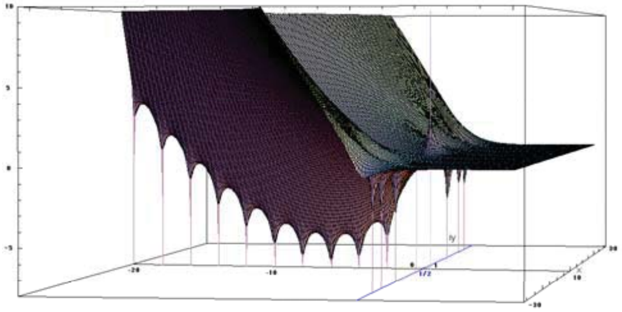

Fig.1 log|ζ(s)| (ζ(s): Riemann zeta function)

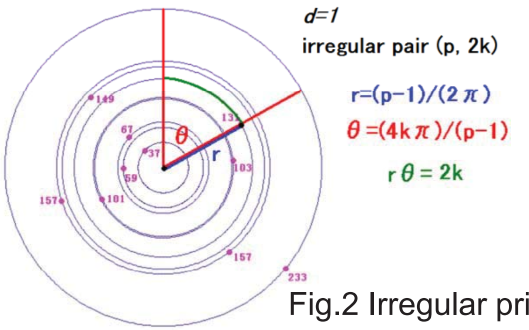

Fig.2 Irregular primes and indices

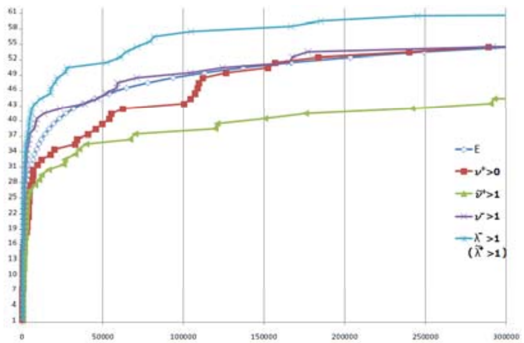

Fig.3 The number of exceptional primes

Content:

The main subject of our research is the ideal class groups of algebraic number fields. We have particularly investigated Greenberg‘s conjecture and Vandiver’s conjecture on the class numbers of real cyclotomic fields by using computers. Furthermore, we are also interested in new applications of algebraic systems such as algebraic number fields and elliptic curves, which have strong connections with cryptography.

A lot of mathematicians have been interested in Riemann zeta function (cf. Fig.1). Its special values have deep relations with the ideal class groups of cyclotomic fields (cf. Fig.2). These relations are expressed as correspondences of the class numbers of real cyclotomic fields and the indices of their circular units in full ones.

Greenberg‘s conjecture states that their p-parts are bounded in the Zp-extension. Moreover, Vandiver's conjecture states that they are trivial for p-cyclotomic fields. We have been studied these conjectures by using arithmetic special elements such as cyclotomic units, Gauss sums, p-adic L-functions and auxiliary prime numbers. As results, we could find a lot of examples for which Greenberg's conjecture holds, and a lot of exceptional prime numbers for the Iwasawa invariants (cf. Fig.3).

Keywords : algebraic number field,
class number, elliptic curve, cryptography

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